5. Given a Brownian motion Wt, when is WtNa martingale? Why Wt3 is not a martingale?

When N=0 or 1, WtN  is a martingale.

When N=3, for any t>s>=0,

E(Wt3- Ws3 | Ws) = E( (Ws+Wt-Ws)3 - Ws3 | Ws)

=E ( 3\*(Wt-Ws)2\*Ws +3\*(Wt-Ws)\*Ws2 +3\*(Wt-Ws)3 | Ws)

= 3\* Ws\*(t-s) + 3\* (E(Wt| Ws)-Ws)\*Ws2+3\*E(Wt-Ws)3

= 3\* Ws\*(t-s)

It is not equal to zero. So E(Wt3| Ws) = Ws3 is not always true. Wt3 is not a martingale.

6. dSt = St \* (mu dt + sigma dWt)

Let f(x) =1/x, then f’(x)=-x^(-2), f”(x)=2x^(-3).

By Ito’s Lemma, d(1/St) = -St^(-2) \* dSt + ½\* 2\* St^(-3) \* d<S>t

= (1/St) \* [ (-mu + sigma^2) dt – sigma dWt ]

Use the Black-Scholes model, the Call option price (1/St)\* N(d1) – K e^(-(T-t)) \* N(d2),

where d1= [ ln(1/ St /K) + (r+ ½ sigma^2) (T-t) ] / (sigma\* sqrt(T-t)) and d2=d1-sigma \*sqrt(T-t).